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14. ABSTRACT

NASCAP-2K can be used to calculate plasmadynamic effects as well as steady-state charging and current collection. In this paper we consider electron dynamics in the sheath of a VLF antenna. We estimate the sheath size, and show 1-D calculations for both sine wave and square wave excitation. The results show strong electrostatic plasma oscillations at the sheath edge. Then we use NASCAP-2K to approach the problem from 2 directions. Run as explicit Particle-in-Cell, NASCAP-2K can duplicate the square wave results through the first maximum in the plasma oscillation, obtaining excellent agreement with the 1-D results. Due to the computational demands of PIC, this approach can only simulate the early transients using a common PC. To learn more about the floating potential of the antenna, NASCAP-2K can be run as hybrid PIC to capture the ion dynamics over the full wave cycle.

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NASCAP-2K Simulation of a Low-Frequency Antenna in Low-Earth Orbit

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Abstract

NASCAP-2K can be used to calculate plasmadynamic effects as well as steady-state charging and current collection. In this paper we consider electron dynamics in the sheath of a VLF antenna. We estimate the sheath size, and show 1-D calculations for both sine wave and square wave excitation. The results show strong electrostatic plasma oscillations at the sheath edge. Then we use NASCAP-2K to approach the problem from 2 directions. Run as explicit Particle-in-Cell, NASCAP-2K can duplicate the square wave results through the first maximum in the plasma oscillation, obtaining excellent agreement with the 1-D results. Due to the computational demands of PIC, this approach can only simulate the early transients using a common PC. To learn more about the floating potential of the antenna, NASCAP-2K can be run as hybrid PIC to capture the ion dynamics over the full wave cycle.

Introduction

There is current interest in generating VLF waves in space for the purpose of controlled experiments with the waves thought to control the trapped electron population (Inan *et al.*, 2003). A transmitting antenna for this purpose would be several inches in diameter, tens of meters long and have bias amplitudes of hundreds of volts, perhaps kilo-volts. It would interact with a large volume of the surrounding plasma, and be a major driver for spacecraft potential. Thus, it is of major interest to be able to simulate dynamically and in 3-D the antenna together with its host spacecraft and the surrounding plasma.

NASCAP-2K (NASA Air Force Spacecraft Charging Analyzer Program 2000, hereafter Nascap-2k) contains the Particle-in-Cell, PIC, features needed to do this type of problem. However, these features have not been fully exercised since the days of pulsed power experiment, SPEAR II (Space Power Experiments Aboard Rockets, Cohen, 1995). In studying the CHAWS experiment (Charging Hazards and Wake Studies, Davis, *et al.*, 1999), ion trajector˙   ˙sities

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and space potentials, and we have developed the ability to do Hyprid-PIC, i.e., PIC ions with barometric electron densities. For the antenna problem we would like to follow both electrons and ions using PIC. It is important to note, however, that Nascap-2k currently only solves Poisson's equation, rather than the full Maxwell equations, so it computes only curl-free, quasi-static fields. Near term development plans include electro-dynamic improvements to Nascap-2k.

In this paper we describe the antenna problem and for baseline parameters, show the solution as calculated by a 1-D PIC code for electron dynamics in the sheath. We then pose a nearly identical problem for solution with Nascap-2k, demonstrate that the solutions agree, and show the graphics. We also use the Hyprid-PIC capability to study the ion dynamics for a full wave cycle.

Statement of Problem

The objective is to simulate the dynamic sheath around a negative thin rod. We avoid positive polarity because the positive half of the antenna will collect copious electrons, so the maximum potential it can reach is determined by numerous unknown factors, such as the relative size of the spacecraft and antenna. We wish to do this with realistic values of plasma density, ion mass, applied voltage, frequency, magnetic field, and spacecraft velocity. The arbitrary directions of the latter two require a three-dimensional code. A one-dimensional (radial) code can handle magnetic field either parallel to the antenna or circumferential (as would be caused by current flowing in the antenna)

Sheath Size Estimate

The sheath (defined as the region from which electrons are excluded) can be quite large, even for a fairly modest potential of about 100 volts. To calculate the sheath size, we specify the electric field at the antenna radius, R_0. We assume that the external space between the antenna and the sheath is filled with ions at ambient density, ρ. The electric field at any radius, r, between the antenna and the sheath is

$$E(r) = \frac{\rho e (r^2 - R_0^2)}{2\varepsilon_0 r} - \frac{a}{r} E(R_0)$$

The sheath condition is $E(R_s)=0$, where R_s is the sheath radius. We then integrate the electric field from R_s to R_0 to determine the corresponding potential. Figure 1 shows the relation between applied potential and sheath radius for a 10 cm diameter antenna. At a density of 10^{12} m^{-3} the sheath radius at 100 V bias is about 15 cm, and grows to nearly a meter at a density of 10^{10} m^{-3}. The calculations to follow assume a density of 3×10^{11} m^{-3}, giving a sheath radius of about 20 cm.
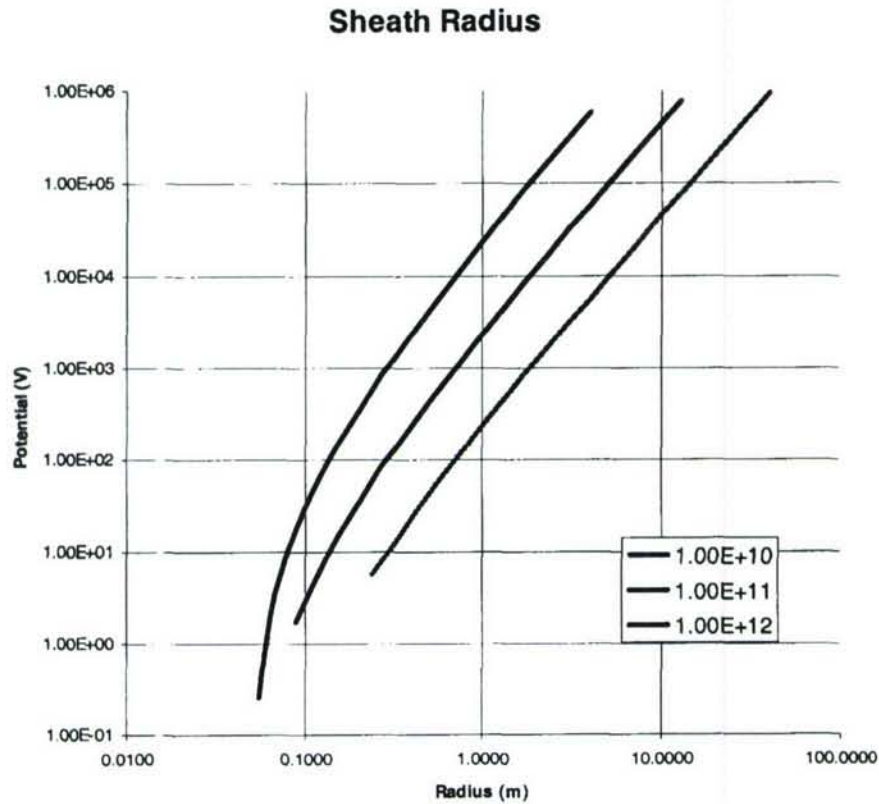
Sheath Radius

Figure 1. Potential vs. sheath radius for negative applied potential on a 10 cm diameter antenna. Curves for three different plasma densities are shown.

Baseline Parameters

Table 1 shows the baseline parameters for the calculation. A density of 3×10^{11} m^{-3} is chosen so that the sheath is large compared with the wire but still very tractable computationally. In general the plasma is cold, so 0.1 eV is used in those places where temperature is required. The antenna frequency is set to 100 kHz, and the process is

Table 1. Parameters for baseline calculations.

Plasma Density	3×10^{11} m^{-3}
Electron Temperature	0.0 or 0.1 eV
Plasma Frequency	3.1×10^7 s^{-1}
Antenna Frequency	100 kHz
Magnetic Field	0.0 or 0.5 gauss
Electron Gyrofrequency	0.0 or 8.8×10^6 s^{-1}
Ion Species	O$^+$

followed for a half-period of 5 μs. To see the effect of magnetic field, a field of 0.5 gauss is chosen. The ordering of the plasma frequency, electron gyrofrequency, and applied frequency is $\omega_p > \omega_c > 2\pi f$.

One-dimensional Calculations

A simple one-dimensional finite element code was written to simulate quasistatic plasmadynamics about a long cylindrical antenna. The computational domain extended out to one meter from an antenna radius of 5 cm, and was divided into 1000 zones in equal increments of r^2. Two ion macroparticles and two electron macroparticles were placed in each zone, with each macroparticle having equal charge. The simulation was run for 2000 timesteps of 2.5 ns each, making up the 5 μs half-period for the 100 kHz frequency. When electrons left the computational space they were replaced as thermal electrons at the boundary.

Figure 2 shows the potential profile at various times in the calculation. As expected, the potential is rapidly screened to about the expected sheath radius as electrons are expelled from the sheath. At certain times a positive potential region appears. This is an effect of electron inertia, as the moving electrons do not stop of their own accord, but must be attracted back towards the sheath boundary. In Figure 3 through Figure 5 we plot (1) the maximum potential at times when a positive region appears; (2) the location of the maximum potential; and (3) the location of the sheath edge, indicated by a sharp drop in the charge density from nearly the ambient ion density to nearly zero.
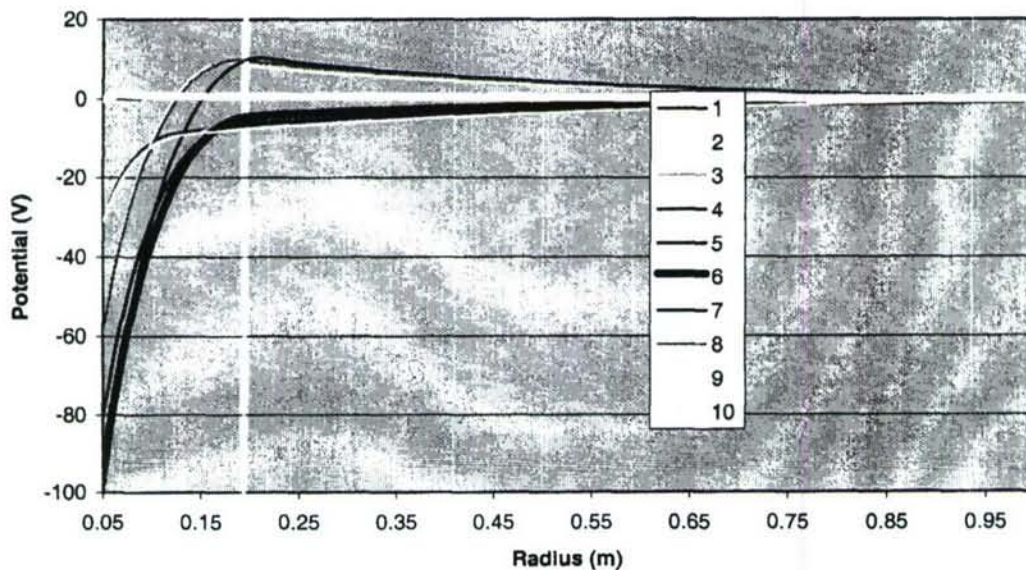

Figure 2. Potential profile at various times during the one-dimenional calculation.
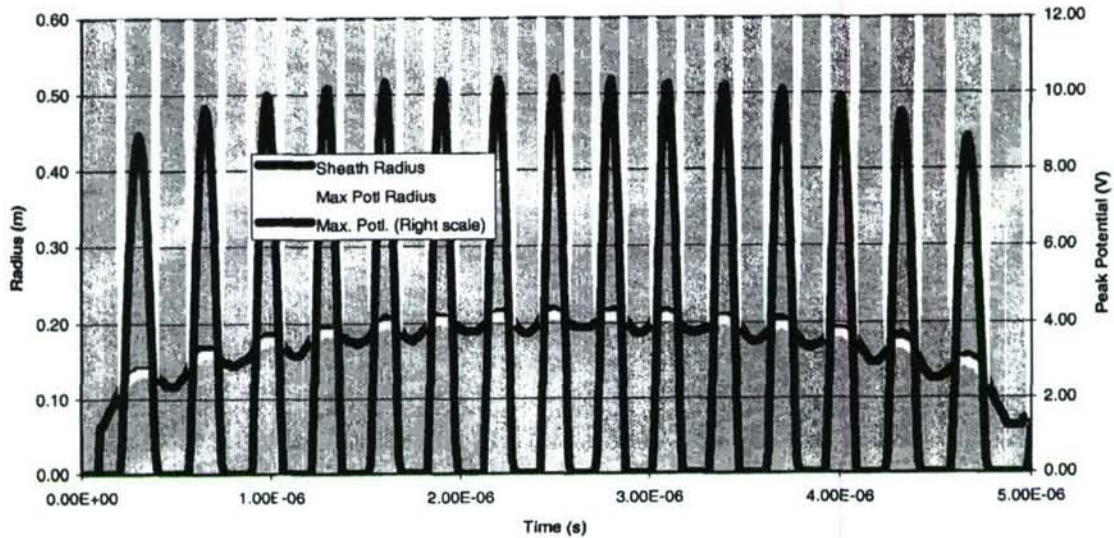
Figure 3. Simulation results for half sine wave and no magnetic field, showing peak positive potential (magenta curve. right scale), location of peak (yellow curve, left scale) and location of sheath edge (dark curve, left scale).
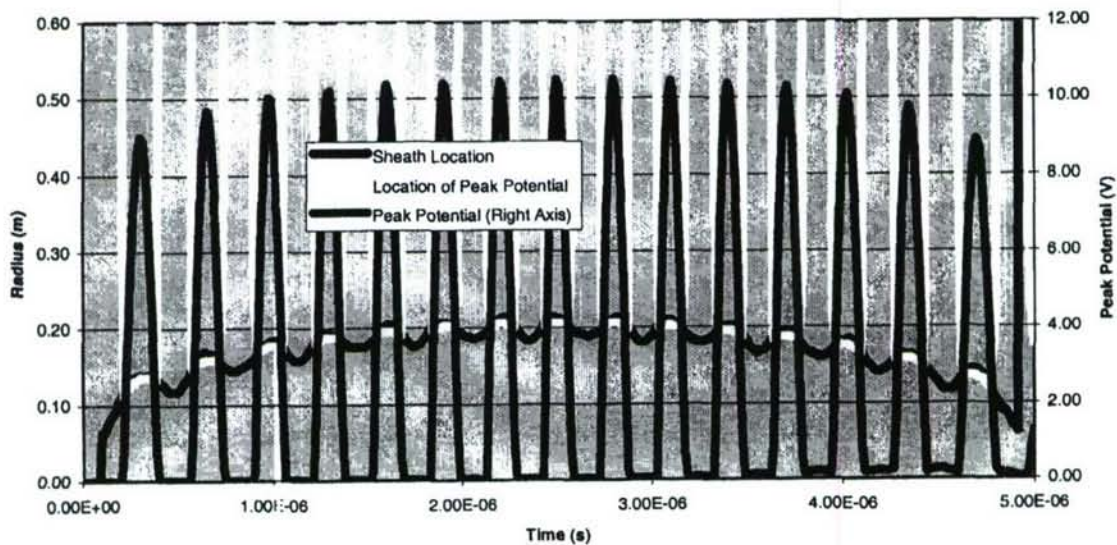

Figure 4. Same as Figure 3, for half sine wave with magnetic field of 0.5 gauss.

Figure 3 and Figure 4 show results for a half sine wave, for which the applied negative potential continuously rises and returns to zero. The magnitude of the potential maximum is 8 to 10 V, and the oscillation frequency is somewhat less than the electron plasma frequency. The sheath edge occurs at a radius of about 20 cm as calculated above, with oscillations of about two cm. The potential maximum, when it occurs, is just inside the sheath edge.
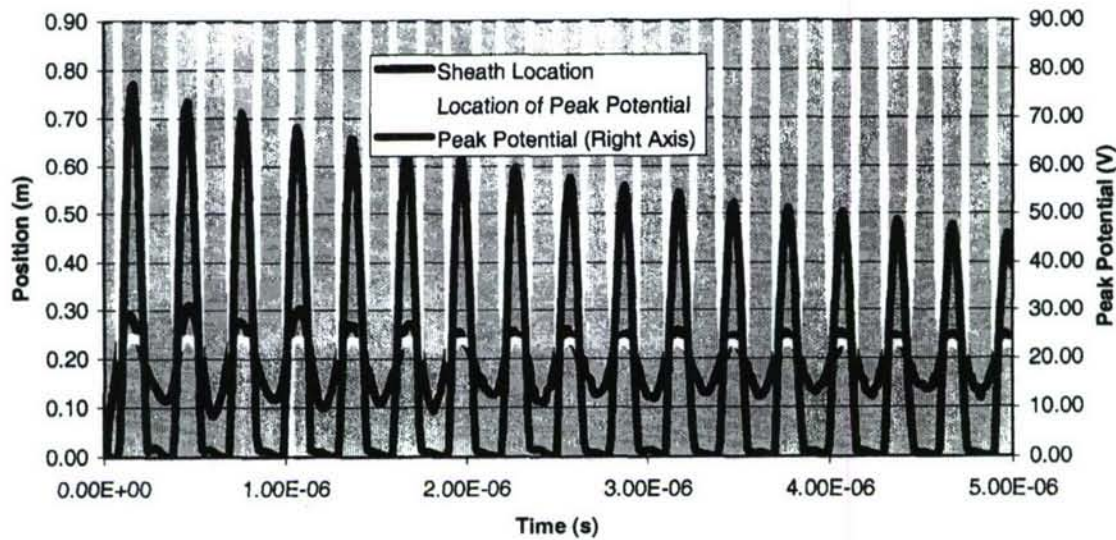
Figure 5. Same as Figure 3, for square wave with 0.5 gauss magnetic field.

Two differences can be noted between the un-magnetized (Figure 3) and magnetized cases. First, in the un-magnetized case the potential goes completely non-positive between peaks, whereas in the magnetized case a positive peak forms inside the outer boundary. This occurs because the magnetic field inhibits inward diffusion of the thermal electrons. Second, in the un-magnetized case the sheath remains at the end of the pulse, even though the potential goes to zero, while in the magnetized case the sheath disappears. This occurs because the magnetic field traps inbound electrons in the sheath region, whereas in the absence of magnetic field inbound electrons collide with the antenna.

Square wave calculations were done in one-dimension to compare with the Nascap-2k calculations below, and are shown in Figure 5. Much stronger plasma oscillations are seen, with the initial oscillation at 75 V. The sheath edge oscillates 10 cm on either side of its average position at 20 cm, and the peak potential occurs well inside the sheath. The 0.5 gauss magnetic field doubles the rate of decay of the oscillations compared to the un-magnetized case (not shown).

Three-dimensional PIC Calculations
Three-dimensional calculations were done with Nascap-2k to demonstrate the feasibility of such calculations. Figure 6 shows the Nascap-2k antenna model embedded in a nested grid. The antenna consists of two square rods, each 10 cm on a side and 4 m long. The outer boundary of the grid is a square 1.32 m on a side. The coarse resolution is 11 cm, with 5.5 cm resolution near most of the antenna, and 2.75 cm resolution in a limited region. Initially, 8 electron macro-particles and 8 ion macro-particles were placed in each zone, positioned so as to represent a uniform charge distribution in the context of the non-linear interpolants. A negative 100 V square wave was applied to half of the antenna, and each timestep consisted of (1) tracking the particles for 2.5 ns, (2) sharing the particle charge to the nodal coefficients in accordance with the nonlinear interpolants, and (3) recalculating the potential in preparation for the next tracking phase.
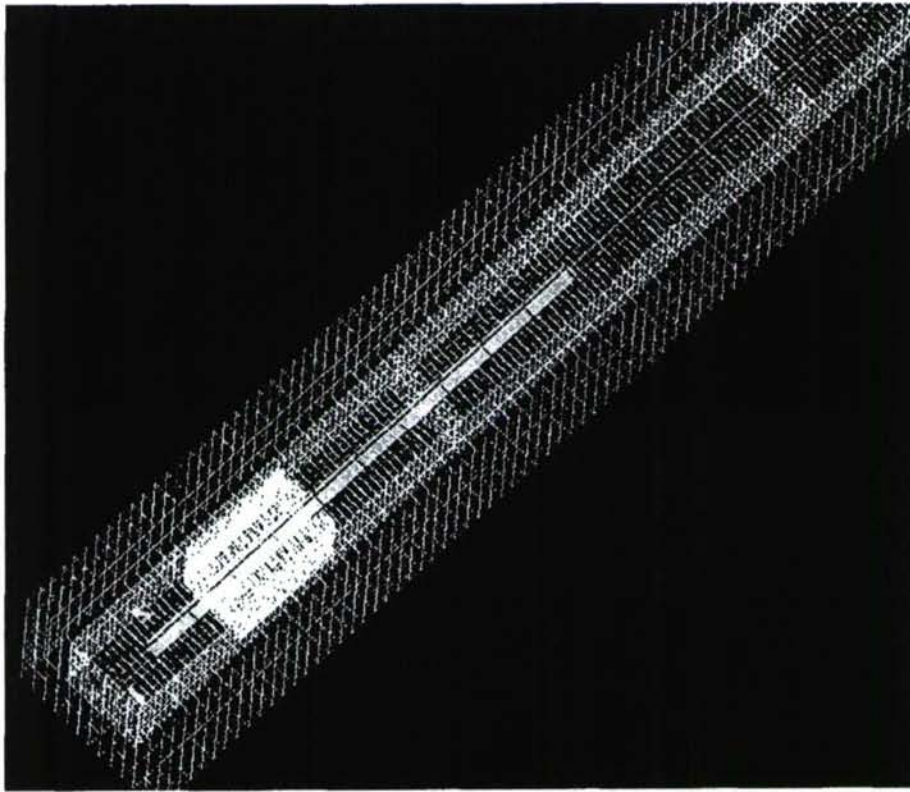
Figure 6. Nascap-2k antenna model, showing antenna and gridding.

The initial state is represented in a three-dimensional view in Figure 7, and a planar view in Figure 8. These figures show the potential in a cut plane, and a plane of nearby electron macro-particles. The ion macro-particles have the same initial configuration, but move negligibly during the simulation time. Note that the apparent high density of particles in the subdivided region is balanced by correspondingly reduced particle weight. Figure 9 show close-ups of the simulation at later times. The left figure shows the electron motion at 50 ns leading to sheath radii of 18 cm. Note the effects of the square cross-section. Particles that started out near the flat, low-field region have moved considerably less than those that started out near the high-field corners. Figure 9-right shows the configuration at time of maximum positive potential (137 ns). At this time, there is a high, broad maximum in the potential, with electrons excluded from a region that extends well beyond the location of the potential maximum.
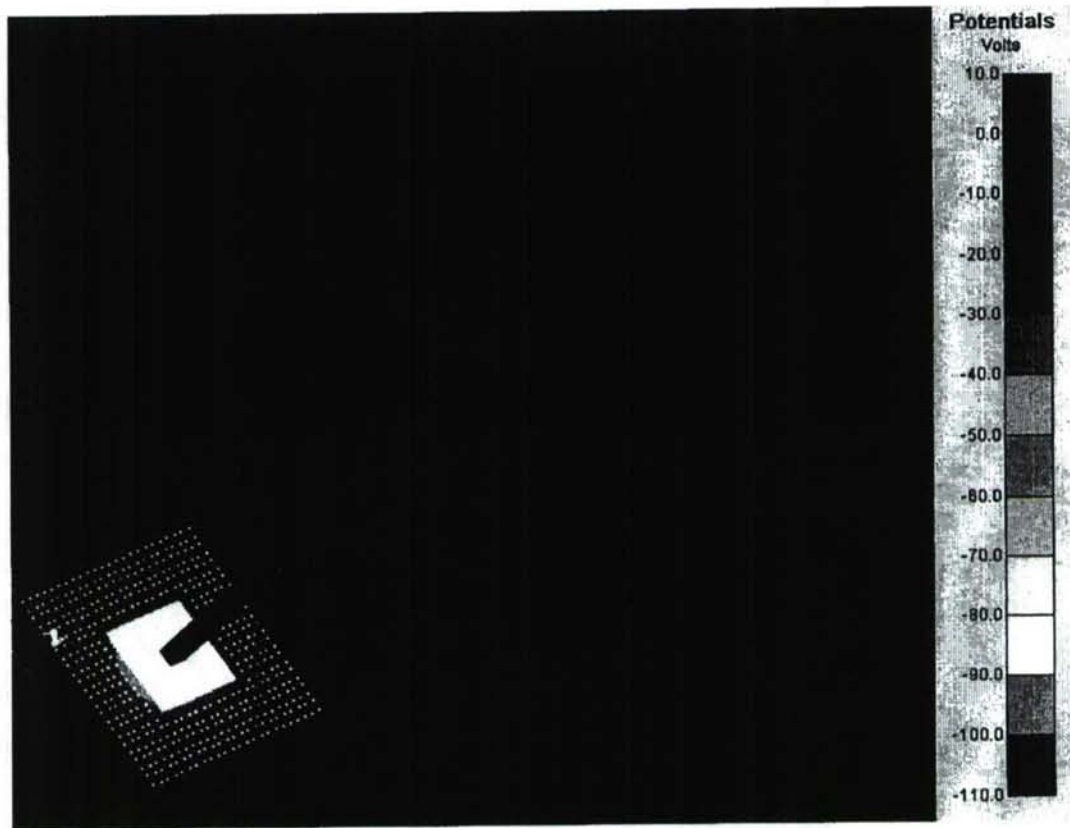
Figure 7. Nascap-2k antenna model showing potentials and particle positions after 2.5 ns.
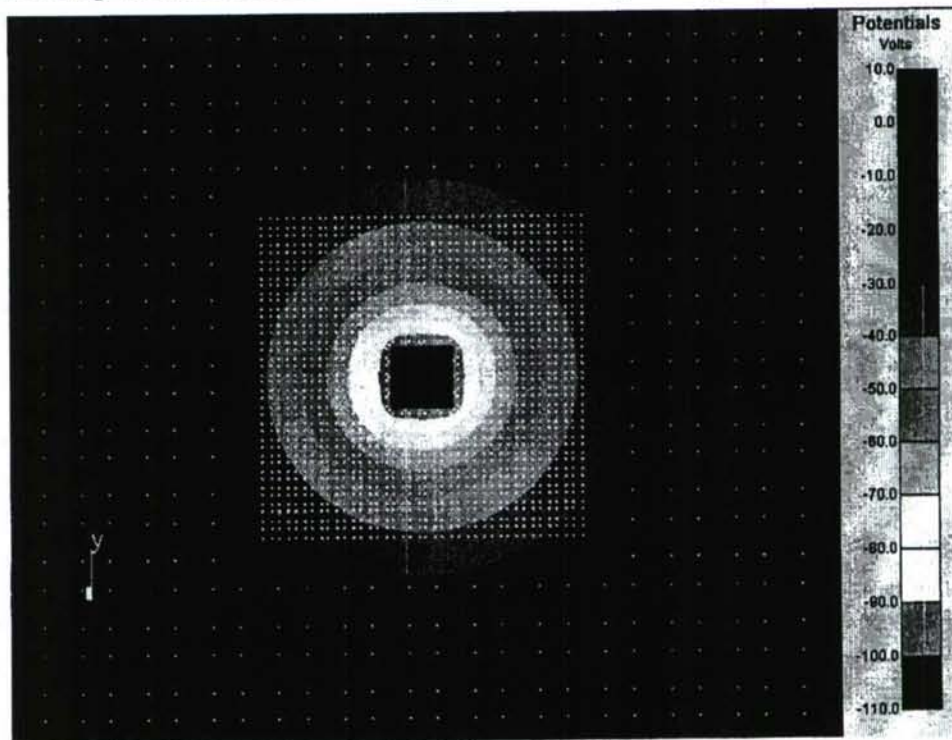

Figure 8. Planar view of initial potentials and particles, as shown in Figure 7.
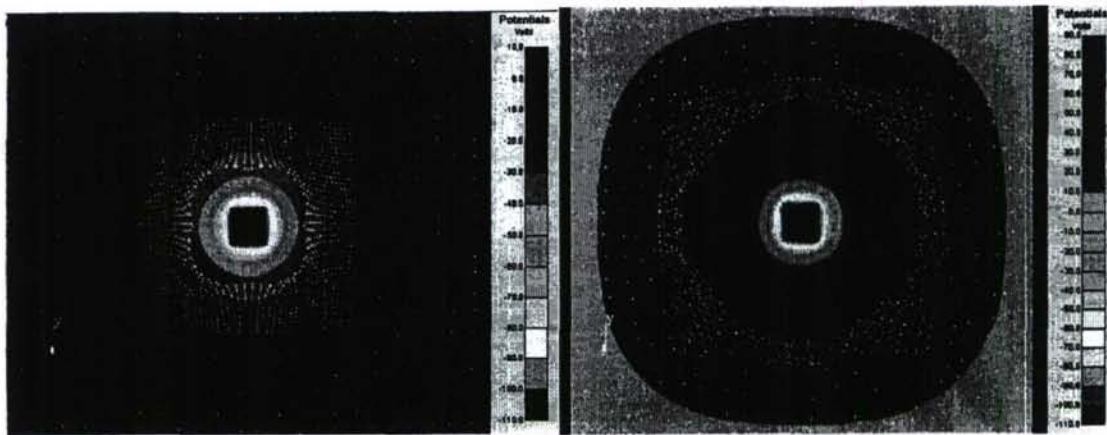
Figure 9. Blowups of particles and potentials. Left: sheath radius of about 18 cm after 50 ns. Right: at time of maximum positive potential (137 ns).

Figure 10 shows another view of the final configuration, with potentials in a plane containing the antenna Note that there is no apparent difference between the potentials in the highly resolved region and in the less resolved region, suggesting that the highest level of resolution may not be needed.
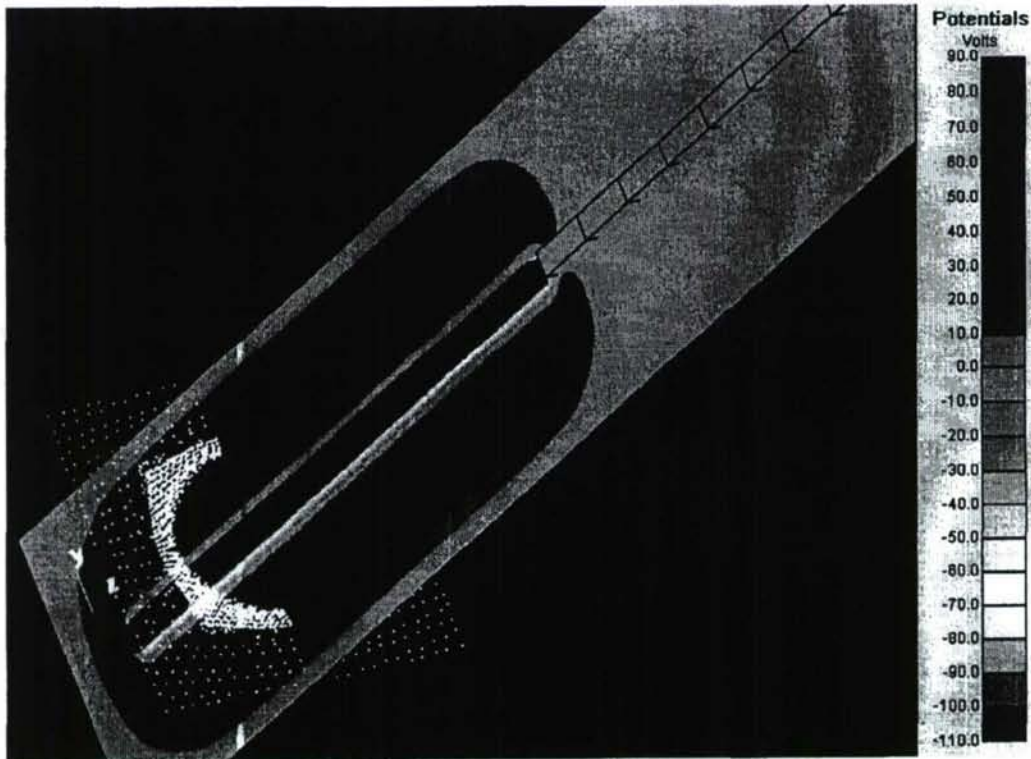

Figure 10. Another view of the final configuration (at 137.5 ns), showing potentials in a plane containing the antenna.

Figure 11 shows the development of the sheath radius and maximum potential calculated by Nascap-2k, compared with the one-dimensional sheath radius result. The maximum potential of 75 V, as well as the time of first maximum (140 ns) is in excellent agreement with the one dimensional result. The maximum sheath radius calculated by Nascap-2k is larger than the one-dimensional result in proportion to the effective larger size of the 10 cm square antenna vs. the 10 cm diameter round antenna.
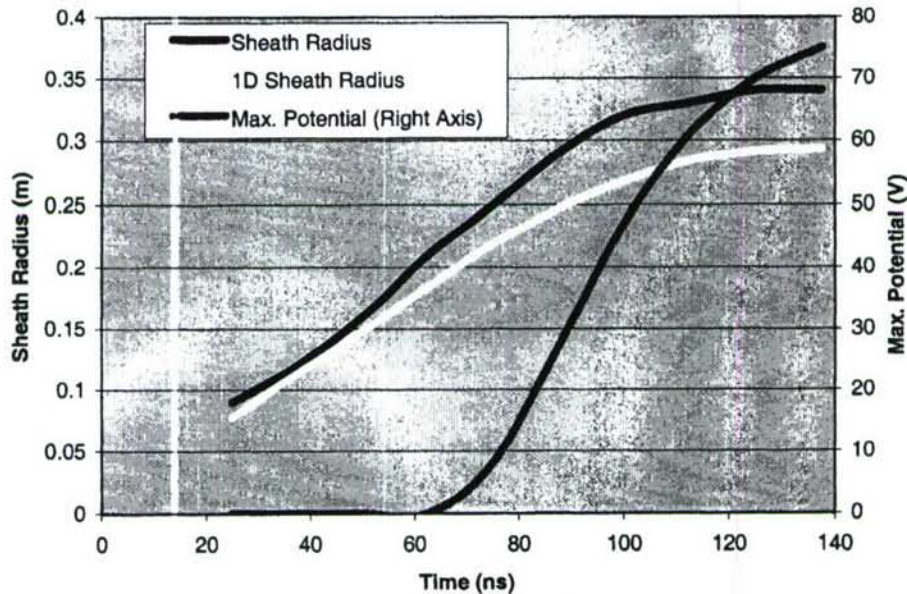

Figure 11. Nascap-2k results for sheath radius (dark curve) and maximum potential (magenta curve, right scale) compared with one-dimensional sheath radius results (yellow curve).

Hybrid-PIC Calculation

We also attempted a more ambitious antenna calculation than above. The intent was to show that a calculation could be performed for parameters more nearly resembling the frequency, waveform, and plasma density of interest for VLF experiments. The old and new calculations are contrasted in Table 2. In both cases, the spacecraft body and positive portion of the antenna were grounded, as static calculations indicated that electron currents would limit the body to negligible positive potentials.

The Hybrid PIC technique is appropriate for low frequency problems where the electrons may be expected to behave as an equilibrium fluid, but where the ions must be treated as a kinetic gas using the PIC method. In this case, we used the simplest electron fluid, the Boltzmann equilibrium, and only the O^+ ions were tracked. The potentials were again solved at each time step. The calculations were carried out to 1.9 cycles of the wave. A snapshot of the 2 kHz calculation is shown in Figure 12. The figure shows the potentials in a cut plane midway through the negative end of the antenna. Although only a representative subset of the ions macro-particles are shown, the figure still illustrate how Nascap-2k can perform these calculation with a relatively low number of particles compared to more common approaches to PIC due to the high order finite element potential

solver. The Nascap-2k potential solver uses non-linear interpolating functions that can guarantee strictly continuous electric fields as well as potential. This dramatically reduces the electric field noise that would otherwise force the calculation to use orders of magnitude more particles and accordingly more times. These simulations took on the order of 1 day on a dedicated GHz PC.

Table 2. Contrasting parameters for the old and new calculations.

| Parameter	PIC Calculation	Hybrid-PIC Calculation
Frequency	100 kHz	2, 6. 20 kHz
Plasma Density	3×10^{11} m^{-3}	1×10^{10} m^{-3}
Plasma Frequency (electron)	4.92 MHz	898 kHz
Plasma Frequency (Oxygen)	28.5 kHz	5.2 kHz
Peak applied potential	-100 V	-100 V
Waveform	Square wave	Sine Wave
Sheath radius	34 cm	60 cm
Timestep	2.5 ns	1/50f
Time Simulated	0.140 μs	2/f
Antenna Length	4 m	10 m
Antenna Radius	0.05 m	0.1 m
Initial No. of Electrons (Ions)	350,000	966,000
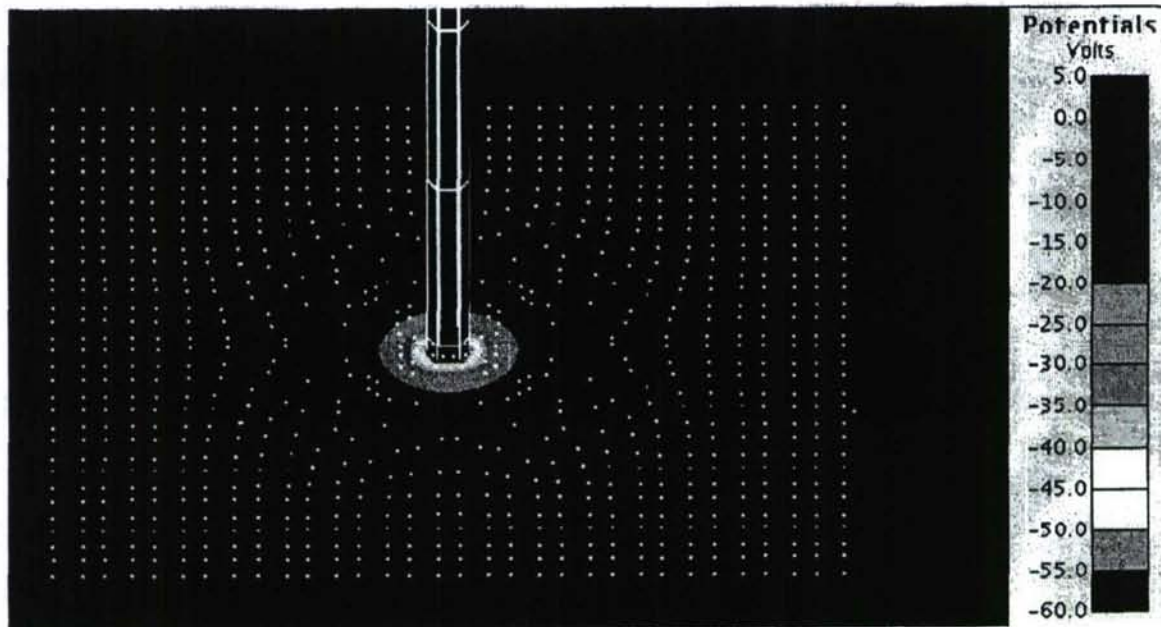
Figure 12. Subset of ion positions and potentials from 2 kHz model at time step 20 at an antenna voltage of –59 Volts.

The calculations do not reached a fully repeating state, but the cycle to cycle variations are not so significant that we cannot draw some preliminary conclusions. Figure 13 shows a summary of the three calculations at frequencies of 2, 6, and 20 kHz. The ion plasma

frequency is 5 kHz. It can be observed that the current lags the voltage by an amount that becomes noticeably significant as the frequency exceeds the ion plasma frequency. We can also observe that ion current profile begins to deviate from the driving sinusoid and appears to be developing harmonics close to the ion plasma frequency.
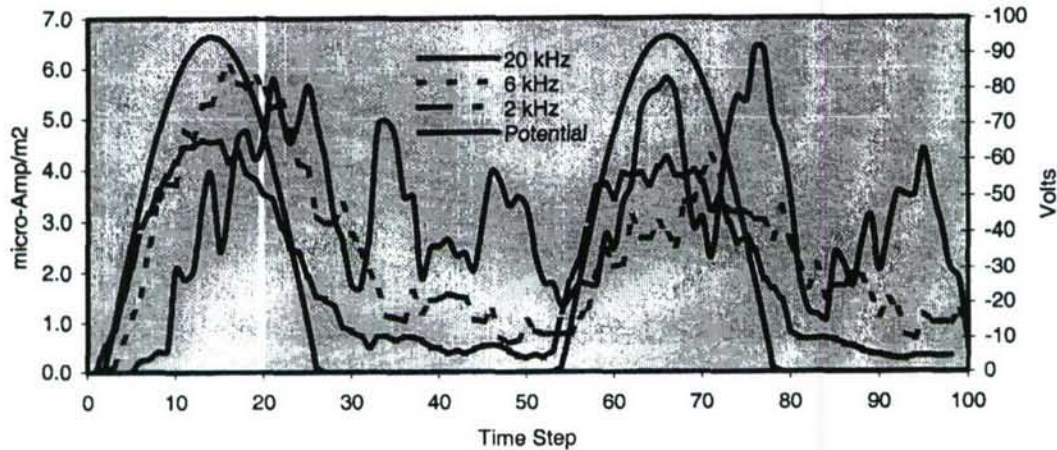

Figure 13. Current and voltage history for the Hybrid PIC calculation.

Conclusions

When the driving waveform contains high frequency transients as with the square wave, electron dynamics are important in the large sheaths of VLF antennas under space conditions. Large electrostatic oscillations are to be expected, producing at times positive potential regions about a negative antenna. The amplitude of the oscillations is waveform dependent. Nascap-2k is shown to have the ability to perform quasi-electrostatic PIC simulations of sheath dynamics in fully three-dimensional geometry. This is important because such simulations can take into account the presence of the host spacecraft and arbitrary directions of magnetic field and spacecraft velocity.

For waveforms with only low frequency content, we have assumed that the electron dynamics can be ignored as a first approximation and have shown that the ion sheath also has interesting dynamics features. These are very preliminary calculations but a tentative conclusion seems to be that without some means to hold the antenna at a more positive potential, the negative potentials will drive ion dynamics that may be rich in harmonic content. Ultimately, predicting the radiation of a low frequency antenna will require following both the electron and ion dynamics as well as a more electro-dynamic treatment of Maxwell's equations. This will drive calculations times, which have been 10s of hours on fast PCs for these simulations, upward by a factor over 100 unless other innovations are introduced. Some of the techniques under evaluation are orbit averaging, where a particle's charge is spread over the full path covered by a large trajectory step; gyro-averaging for the electron orbits, and of course parallel processing.

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